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IRREGULAR INTERACTION OF A MOVING SHOCK WAVE WITH A TANGENTIAL DISCONTINUITY

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K. A. BEZHANOV
(Moscow)

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A linear formulation is used to study irregular interaction of a moving shock wave with a surface of discontinuity separating a gas from a compressible liquid occupying a part of the lower half-space bounded by a rectilinear inclined wall. The problem of interaction in the case when the shock wave in a gas overtakes weak perturbations in the liquid and the flow behind the shock wave is subsonic, was studied in [1].

1. Formulation of the problem. The front of a plane shock wave moves along a smooth wall at a constant velocity V and emerges, at the instant $t = 0$, on the free surface of a compressible liquid. We investigate the flow in the gas, and in the liquid at $t > 0$. The problem is self-similar and is studied in the linear formulation since the shock wave front moves at a high velocity and the parameter $\varepsilon = R_1 / R_2'$ (R_1 is the gas density behind the shock wave and R_2' is the density of the liquid) is small. The problem is divided into three distinct problems which are solved in the following order.

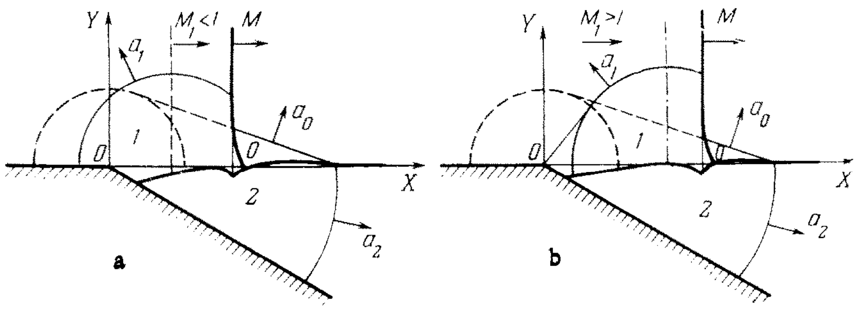

Fig. 1

(a) The problem of motion or liquid acted upon by an unperturbed pressure P behind the shock wave in the gas. Solution of this problem enables us to determine the form of the perturbed tangential discontinuity where $y = \varepsilon f(x)$

$$x = X / (a_2 t), \qquad y = Y / (a_2 t) \tag{1.1}$$

are the self-similar coordinates (see Fig. 1).

(b) When the weak perturbations in the liquid propagate faster than the shock wave in the gas, a region of perturbed flow exists ahead of the shock wave. The parameters of this flow are determined from the perturbations in the tangential discontinuity and in the linear formulation are independent of the gas flow behind the shock wave.

(c) The flow in the diffraction region is determined from the known form of the tangential discontinuity and from the gas flow parameters in the neighborhood of the diffraction region.

In each of the above regions the pressure perturbation satisfies the wave equation

$$p_{XX} + p_{YY} - a^2{}_i p_{tt} = 0 \quad (i = 0, 1, 2) \tag{1.2}$$

and, in the coordinate system moving at the velocity V_1 of flow behind the shock wave, a_i denotes the speed of sound and i indicates the region of flow (see Fig. 1).

Since the number of parameters is large and the problems listed above are solved in a strict sequence, the index accompanying the dimensionless parameters is, in general, omitted.

2. Solution of the problem in the liquid. When $V \leqslant a_2$, the region of perturbed flow in the liquid is bounded by the inclined wall, the arc of the Mach circle with the center at the corner point of the wall, and by the tangential discontinuity. Passing first to the self-similar coordinates (1.1) and then to the polar coordinates and applying the Chaplygin transformations

$$r = \frac{2\rho}{1+\rho^2}, \quad \theta = \operatorname{arctg}\frac{y}{x}, \quad r = \sqrt{x^2+y^2} \tag{2.1}$$

we transform Eq. (1.2) into a Laplace equation in the region $\{\rho < 1, \ -\beta < \theta < 0\}$. After this we can write the pressure as the real part of the analytic function

$$\Phi(\zeta) = p + i\varphi, \quad \zeta = \xi + i\eta = \rho \exp i\theta \tag{2.2}$$

where φ is a harmonic function conjugate to p.

Just as in the theory of conic flows [2], we can obtain the following expression for the complex velocity $W = u + iv$:

$$W = \frac{1}{2}\int \zeta d\Phi + \frac{1}{\zeta} d\Phi, \quad u \to \frac{u}{a_2}, \quad v \to \frac{v}{a_2} \tag{2.3}$$

(here and henceforth the arrows indicate an equality to within the notation used). Then the boundary conditions for the function $\Phi(\zeta)$ have the form

$$p = \begin{cases} \vartheta(\xi_0 - \xi)P, & 0 < \xi < 1, \quad \eta = 0, \\ 0, & -\beta < \theta < 0, \quad \rho = 1, \end{cases} \qquad \xi_0 = \frac{1-\sqrt{1-x_0^2}}{x_0}$$

$$\varphi = 0, \quad 0 < \rho < 1, \quad \theta = -\beta, \quad p \to p/a_2^2 R_2$$

where ϑ denotes a unit Heaviside function and $x_0 = V / a_2$ is the point at which the pressure becomes discontinuous.

Mapping conformally the sector $\{\rho < 1, \ -\beta < \theta < 0\}$ of the ζ-plane onto the upper half of the plane $\omega = \tau + i\sigma, \ \alpha = \pi / \beta$

$$\omega = \left(\frac{1-\zeta^\alpha}{1+\zeta^\alpha}\right)^2, \quad (\zeta^\alpha = \xi^\alpha \text{ when } \zeta = \xi > 0) \tag{2.4}$$

and carrying out the substitution $\Phi(\omega) = \sqrt{1-\omega}\,\Phi_1(\omega)\,(\sqrt{1-\omega} = 1$ when $\omega = 0)$, we obtain the Dirichlet problem for the function

$$p_1 = \frac{\vartheta[(1-\tau)(\tau-\tau_0)]}{\sqrt{1-\tau}}P, \quad -\infty < \tau < \infty, \quad \tau_0 = \left(\frac{1-\xi_0^\alpha}{1+\xi_0^\alpha}\right)^2$$

Writing a solution of this problem in the form of the Schwartz integral [3], we obtain the analytic function

$$\Phi(\omega) = \frac{iP}{\pi} \ln \frac{\sqrt{1-\omega} - \sqrt{1-\tau_0}}{\sqrt{1-\omega} + \sqrt{1-\tau_0}} \quad (\operatorname{Re}\Phi(\omega) = 0 \quad \text{for } \omega = \tau < \tau_0) \qquad (2.5)$$

The distribution of the vertical velocity component at the interface we find from (2.3) −(2.5)

$$v = \int_x^1 \frac{\sqrt{1-s}}{s}\, d\varphi, \quad x = \frac{2\xi}{1+\xi^2} \qquad (2.6)$$

$$\varphi = \frac{P}{\pi} \ln \frac{\left| \sqrt{\xi_0^{\alpha}}\,(1+\xi^{\alpha}) - (1+\xi_0^{\alpha})\sqrt{\xi^{\alpha}} \right|}{\sqrt{\xi_0^{\alpha}}\,(1+\xi^{\alpha}) + (1+\xi_0^{\alpha})\sqrt{\xi^{\alpha}}}$$

The form of the tangential discontinuity is determined from the solution of the differential equation

$$xf'(x) - f(x) = -v(x), \quad f(1) = 0 \qquad (2.7)$$

which is obtained from the condition of kinematic compatibility at the interface. From (2.7) it follows that $f'(1) = 0$, i.e. the Mach circle wave does not effect the smoothness of the interface at the point $x = 1$.

In a number of cases the distribution of the vertical velocity component at the interface and the form of the tangential discontinuity can both be obtained in explicit form. For example, for $x_0 \leqslant 1$ and $\beta = \pi / 4$ we have

$$v = \frac{P}{\pi x_0}\left[2\operatorname{arctg}\frac{x}{x_0\sqrt{1-x^2}} - \sqrt{1-x_0^2}\ln\frac{|x^2 - x_0^2|}{(x_0\sqrt{1-x^2} + \sqrt{1-x_0^2}x)^2} - \pi\right]$$

$$f(x) = \frac{2P}{\pi x_0}\left\{\operatorname{arctg}\frac{x}{x_0\sqrt{1-x^2}} + \frac{2-x_0^2}{x_0}x\ln\frac{1+\sqrt{1-x^2}}{x} + \frac{\sqrt{1-x_0^2}}{2x_0}\left[(x-x_0)\ln|x^2 - x_0^2| + 2\ln(x_0\sqrt{1-x^2} + \sqrt{1-x_0^2}x) - 2x\ln\frac{(\sqrt{1-x_0^2} + \sqrt{1-x^2})(x_0^2 + (1-x_0^2)x^2)}{1+\sqrt{1-x_0^2}\sqrt{1-x^2}}\right] - \frac{\pi}{2}\right\}$$

In what follows, we shall express the solution of the problem in the gas in terms of the second derivative of the function defining the form of the tangential discontinuity and given in explicit form for all values of the parameters β and x_0 by (2.6) and (2.7). When $x = x_0 < 1$, the function v will have a logarithmic singularity and $f(x)$ will have a corner point just as in the case of an incompressible fluid, while at $x_0 \geqslant 1$ the singularity will vanish. Study of the flow of an incompressible fluid near the point of pressure discontinuity, using a nonlinear formulation, shows that the perturbed interface has a spiral-like form [4,5]. The interface always departs from the corner point of the wall.

When $V > a_2$, a weak wave tangent to the Mach circle arc emerges from the point of intersection of the shock wave with the tangential discontinuity. The flow contained within the region bounded by the weak wave, the Mach circle arc and the interface, has constant parameters determined by the relations at the weak wave. The form of the interface within this region is represented, in accordance with (2.7), by an inclined straight line which merges smoothly at the point $x = 1$ with the form of the interface obtained for $x < 1$.

3. Gas flow outside the diffraction region. We determine the gas flow in zones adjacent to the region of diffraction using the functional-invariant method of Smirnov and Sobolev [6].

The perturbed region ahead of the shock wave is bounded by the bow Mach wave emerging from the point $x = k^{-1}$ at the angle π arc sin k to the unperturbed tangential discontinuity, and $k = a_0 / a_2$. Passing in (1.2) first to the self-similar variables $x = X / (a_0 t)$, $y = Y / (a_0 t)$ and then to the polar coordinates, and applying the transformations

$$\mu = \arccos r^{-1}, \quad r > 1 \tag{3.1}$$

we obtain the wave equation

$$p^\circ_{\mu\mu} - p_{\theta\theta}^\circ = 0, \quad p^\circ \to p^\circ / (a_0^2 R_0) \tag{3.2}$$

The condition that the vertical velocity components of the gas and the liquid are equal at the interface is obtained, with the help of the linearized equation of motion (the velocity v is known from (2.6)), in the form

$$p_\theta^\circ (r, 0) = r^2 v' (kr) \tag{3.3}$$

The characteristics of (3.2) touch the Mach circle $r = 1$ since the perturbations cannot propagate upstream, the solution of the problem (3.2),(3.3) has the form

$$p^\circ = \int_{\mu_0}^{\mu+\theta} v' (k \sec s) \sec^2 s ds, \quad \mu_0 = \text{arc} \cos k \tag{3.4}$$

The flow is irrotational in the region under consideration, therefore the equations determining the vertical component of the velocity v^0 satisfies an equation analogous to (3.2). The condition that the vertical components of the gas and the liquid velocities are equal at the interface enables us to determine v^0 over the whole region of perturbed flow

$$v^\circ = v \left(\frac{kr}{\cos\theta - \sqrt{r^2 - 1} \sin\theta} \right), \quad v^\circ \to \frac{v^\circ}{a_0} \tag{3.5}$$

Using the linearized equation of motion written in polar coordinates, we can find the horizontal velocity component ($r(\theta)$ is the equation of the bow Mach wave)

$$u^\circ = \int_{r(\theta)}^{r} v' \left(\frac{ks}{\cos\theta - \sqrt{s^2 - 1} \sin\theta} \right) \frac{ds}{(\cos\theta - \sqrt{s^2 - 1} \sin\theta) \sqrt{s^2 - 1}} \tag{3.6}$$

$$r(\theta) = k^{-1} \cos \mu_0 \sec (\theta - \mu_0), \quad u^\circ \to u^\circ / a_0$$

When the flow behind the shock wave is supersonic, the corner point of the wall lies outside the diffraction region and an additional region of perturbed motion exists bounded by the tangential discontinuity, the characteristic emerging from the corner point of the wall and an arc of the Mach circle the center of which moves at the velocity V_1. Passing to the self-similar coordinates

$$x = (X - V_1 t) / (a_1 t), \quad y = Y / (a_1 t) \tag{3.7}$$

and application of the transformation (3.1) transforms (1.2) into (3.2). This, together with the boundary condition

$$p_\theta (r, \pi) = r^3 f'' (M_1 - r), \quad p \to p / (a_1^2 R_1), \quad M_1 = V_1 / a_1$$

makes possible the determination of the pressure

$$p = \int_{\mu_1 - \pi}^{\mu - \theta} f'' (\sec s + M_1) \sec^3 s \, ds, \quad \mu_1 = \arccos M_1^{-1} \tag{3.8}$$

4. Formulation of the boundary conditions in the diffraction region. The diffraction region in the gas is bounded by the shock wave, the Mach circle arc, and by the tangential discontinuity at $M_1 > 1$, or by the solid wall and the tangential discontinuity at $M_1 < 1$.

When $M_1 < 1$, a zone of nonlinear flow exists near the corner of the wall. The zone is generated by the departure of the interface from the wall corner and can be replaced be the action of a dipole. In mathematical terms it means that the second derivative of the boundary has two delta functions. These delta functions have finite densities of opposing signs

$$-\mathrm{tg}\, \beta \delta (x + M_1) + (\mathrm{tg}\, \beta + O(\varepsilon)) \delta (x + M_1 + O(\varepsilon))$$

and tney define, as $\varepsilon \to 0$, a dipole with momentum density of $\varepsilon \, \mathrm{tg}\, \beta$ [7]. In this case the boundary condition at the rigid wall and on the tangential discontinuity will have the form

$$\begin{aligned} p_y (x, 0) = {} & \vartheta (1 - M_1) \varepsilon \, \mathrm{tg}\, \beta M_1 \delta' (x + M_1) - \\ & [\vartheta (1 - M_1) \vartheta (x - x_1) + \vartheta (M_1 - 1)] \, x^2 f'' (x + M_1) \end{aligned} \tag{4.1}$$

(x_1 is the abscissa of the point of intersection of the interface with the inclined wall)

The boundary condition at the arc of the Mach circle $\{r = 1, \ \theta_1 < \theta < \pi\}$ is obtained from (3.8) with the help of the formula for changing the variable in the delta function derivative [8] (γ is the ratio of specific heats)

$$\begin{aligned} p_\theta (1, \theta) = {} & \vartheta (M_1 - 1) [\nu_1 \delta' (\theta - \theta_2) + \nu_2 \delta (\theta - \theta_2) - \\ & \vartheta (\theta - \theta_2) \sec^3 \theta f'' (\sec \theta + M_1)] \end{aligned}$$

$$\theta_1 = \mathrm{arc\,tg}\, (m_1 m^{-1}), \quad \theta_2 = \pi - \mu_1, \quad m_1 = \sqrt{1 - m^2}, \quad M = V / a_0$$

$$m = \sqrt{\frac{2 - (1 - \gamma) M^2}{2 \gamma M^2 + 1 - \gamma}}, \quad \nu_1 = \frac{\varepsilon \, \mathrm{tg}\, \beta M_1}{M_1^2 - 1}, \quad \nu_2 = \frac{M_1^2 - 2}{\sqrt{M_1^2 - 1}} \nu_1$$

where, as in (4.1), the zone of nonlinear flow is replaced by the action of a dipole. The boundary condition at the shock wave $\{x = m, \ 0 < y < m_1\}$ has the form

$$m_1^2 p_x + [mBy^{-1} - (m + A)y]p_y = \vartheta(y_0 - y)F(y)$$
$$F(y) = (Cy + Ey^{-1})p_y^\circ + (Dy + Gy^{-1})v_x^\circ - \lambda^{-1}my\vartheta_y^\circ \qquad (4.2)$$

and the right-hand side of (4.2) is obtained from (3.4) and (3.5).

$$p_y^\circ = [y + Mkn(y)](M^2 + \lambda^2 y^2)K(y), \quad n(y) = \sqrt{M^2 - 1 + \lambda^2 y^2}$$
$$v_x^\circ = [n(y)(M^2 - \lambda^2 y^2) + M\lambda y(M^2 - 2 - \lambda^2 y)]\, K(y)$$
$$v_y^\circ = [2M\lambda yn(y) + \lambda^2 y^2 + M^2(\sqrt{M^2 + \lambda^2 y^2} - 1)]\, K(y)$$
$$K(y) = \frac{\lambda k}{n(y)[M - \lambda yn(y)]^2}\, v'\left[\frac{k(M^2 + \lambda^2 y^2)}{M - \lambda yn(y)}\right]$$
$$A = \frac{M^2+1}{2mM}, \quad B = \frac{\gamma+1}{2}\, \frac{M^2-1}{(\gamma-1)M^2+2}, \quad C = A - \frac{\gamma-1}{2} M_1,$$
$$D = \frac{\gamma+1}{2}\, \frac{M}{1-M^2}$$
$$E = (M^2 + 1 - m\lambda M)\frac{m}{\lambda^2}, \quad G = (m\lambda - 2M)\frac{m}{\lambda^2},$$
$$y_0 = \frac{1 - kM}{k\lambda\sqrt{\lambda^2 - 1}}, \quad \lambda = \frac{a_1}{a_0}$$

5. Reduction to a boundary value problem for the upper half-plane. Passage to the self-similar coordinates (3.7) followed by a change to polar coordinates and application of the transformation (2.1), converts the equation (1.2) into a Laplace equation in the ζ-plane. The diffraction region has a corresponding curvilinear triangle bounded by the arc of the circle $\{2\rho\cos\theta = m(1 + \rho^2),\ 0 < \theta < \theta_1\}$ of radius $m_1 m^{-1}$ with the center at the point m^{-1}, the arc of the circle $\{\rho = 1,\ \theta_1 < \theta < \pi\}$ and the segment $\{-1 < \xi < (1 - m_1)m^{-1},\ \eta = 0\}$ of the real axis.

The boundary condition for the normal and tangential components of the pressure derivatives has the form (n denotes the inward normal)

$$ap_n + bp_s = c$$
$$a = 1, \quad b = b(\theta), \quad c = F_1(\theta), \qquad 0 < \theta < \theta_1$$
$$a = 0, \quad b = 1, \qquad c = F_2(\theta), \qquad \theta_1 < \theta < \pi$$
$$a = 1, \quad b = 0, \qquad c = F_3(\xi), \quad -1 < \xi < (1 - m_1)m^{-1}$$

$$F_1(\theta) = \vartheta(\theta - \theta_0)\frac{m\sec\theta}{m_1\rho}F(m\operatorname{tg}\theta), \quad b(\theta) = \frac{B\operatorname{ctg}\theta - mA\operatorname{tg}\theta}{\sqrt{1 - m^2\sec^2\theta}}$$
$$F_2(\theta) = \vartheta(M_1 - 1)[\nu_1\delta'(\theta - \theta_2) + \nu_2\delta(\theta - \theta_2) -$$
$$\vartheta(\theta - \theta_2)\sec^3\theta f''(\sec\theta + M_1)$$

$$F_3(\xi) = \vartheta(1 - M_1)\Big\{\nu_3\delta'(\xi - \xi_2) + \nu_4\delta(\xi - \xi_2) -$$
$$[\vartheta(1 - M_1)\vartheta(\xi - \xi_1) + \vartheta(M_1 - 1)]\frac{8\xi^2}{(1+\xi^2)^3} f''\left(\frac{2\xi}{1+\xi^2} + M_1\right)\Big\}$$

$$\theta_0 = \operatorname{arctg}\frac{y_0}{m}, \quad \xi_1 = \frac{1 - \sqrt{1 - x_1^2}}{x_1}, \quad \xi_2 = \frac{\sqrt{1 - M_1^2} - 1}{M_1}$$
$$\nu_3 = \frac{\varepsilon\, 2\operatorname{tg}\beta}{\sqrt{1 - M_1^2}}\, \frac{\xi_2^2}{\xi_2^2 - 1}, \quad \nu_4 = \varepsilon\operatorname{tg}\beta\xi_2\frac{1 + \xi_2^4}{(1 - \xi_2^2)^3}$$

Let us map the region bounded by the curvilinear triangle on the ζ -plane onto the upper half-plane of the plane $\omega = \tau + i\sigma$

$$w = \zeta_1\left(i - \frac{2m_1}{\zeta - \zeta_1}\right), \quad \omega = \frac{1}{2}\left(w^2 + \frac{1}{w^2}\right), \quad \zeta_1 = m + im_1$$

We introduce the function $\Gamma(\omega) = p_\sigma + ip_\tau$ regular in the upper half-plane. This yields the following Hilbert problem in the class of generalized functions [9, 10]:

$$[\vartheta(1-\tau^2) + \vartheta(\tau-1)\sqrt{\tau-1}]\,p_\sigma + [\vartheta(-\tau-1) - \vartheta(\tau-1)\times (N\tau - L)]p_\tau = c(\tau), \qquad -\infty<\tau<\infty$$

$$c(\tau) = \vartheta(-\tau-1)c_1(\tau) + \vartheta(1-\tau^2)c_2(\tau) - \vartheta(\tau-1)\,c_3(\tau)$$

$$c_1(\tau) = \vartheta(M_1-1)\,[\nu_5\delta'(\tau-\tau_1) + \nu_2\delta(\tau-\tau_1)] - \frac{\vartheta(M_1-1)\,\vartheta(\tau-\tau_1)\,\Lambda(\tau)}{\sqrt{\tau^2-1}\,(\sqrt{-\tau+\sqrt{\tau^2-1}} + \sqrt{-\tau-\sqrt{\tau^2-1}} - 2m)}$$

$$c_2(\tau) = \vartheta(1-M_1)\,[\nu_6\delta'(\tau-\tau_2) + \nu_4\delta(\tau-\tau_2)] - \frac{m_1}{\sqrt{2}}\,\frac{\vartheta(1-M_1)\,\vartheta(\tau-\tau_3) + \vartheta(M_1-1)}{\sqrt{1-\tau^2}\,(\sqrt{2}m - \sqrt{1-\tau})}\,\Lambda(\tau)$$

$$c_3(\tau) = \frac{\vartheta(\tau_4-\tau)}{2(\tau+1)}\sqrt{\sqrt{\tau+1} - \sqrt{\tau-1}}\;F\left(m_1\sqrt{\frac{\tau-1}{\tau+1}}\right)$$

$$\Lambda(\tau) = q^3(\tau)\,f''(q(\tau) + M_1), \quad q(\tau) = \frac{\sqrt{2}m - \sqrt{1-\tau}}{\sqrt{2} - m\sqrt{1-\tau}}$$

$$N,\ L = \frac{m_1^2 A \mp mB}{\sqrt{2}m_1^2}, \quad \tau_1 = 1 - \frac{2M^2}{(1-m)^2 M^2}$$

$$\tau_2 = 1 - \frac{2M^2}{(1+mM_1)^2}, \quad \tau_3 = 1 - 2\left(\frac{m-x_1}{1-mx_1}\right)^2, \quad \tau_4 = \frac{m_1^2 + y_0^2}{m_1^2 - y_0^2}$$

$$\nu_5 = \sqrt{\tau_1^2-1}\,(\sqrt{-\tau_1+\sqrt{\tau_1^2-1}} + \sqrt{-\tau_1-\sqrt{\tau_1^2-1}} - 2m)\frac{\nu_1}{m_1}$$

$$\nu_6 = \sqrt{2(1-\tau_2^2)}\,(\sqrt{2} + m_1\sqrt{1+\tau_2} - m\sqrt{1-\tau_2})\,\frac{\nu_3}{m_1}$$

Substitution $\Gamma_1^+(\omega) = \sqrt{\omega^2-1}\;\;\Gamma(\omega)$ ($\sqrt{\omega^2-1} = \sqrt{\tau^2-1}$ with $\omega = \tau > 1$), $\Gamma_1^+(\omega) = p_\sigma^1 + ip_\tau^1$ enables us to pass to the Hilbert problem with continuous coefficients

$$\vartheta(\tau-1)\ \sqrt{\tau-1}\,p_\sigma^1 + (N\tau - L)\,p_\tau^1 = d(\tau), \quad -\infty<\tau<\infty$$

$$d(\tau) = \vartheta(1-\tau^2)\sqrt{1-\tau^2}\,c_2(\tau) + \sqrt{\tau^2-1}\,[\vartheta(\tau-1)\,c_3(\tau) - \vartheta(-\tau-1)(N\tau-L)\,c_1(\tau)]$$

The corresponding Riemann problem has the form [11, 12]

$$\Gamma_1^+(\tau) = G(\tau)\,\Gamma_1^-(\tau) + g(\tau), \quad \Gamma_1^+(\omega) = \overline{\Gamma_1^-(\overline{\omega})}, \quad -\infty<\tau<\infty$$

$$G(\tau) = \frac{N\tau - L - i\vartheta(\tau-1)\sqrt{\tau-1}}{N\tau - L + i\vartheta(\tau-1)\sqrt{\tau-1}}$$

$$g(\tau) = \frac{i2d(\tau)}{N\tau - L + i\vartheta(\tau-1)\sqrt{\tau-1}}$$

We can neglect $\vartheta(\tau-1)$ appearing in the numerator and denominator of the expression for $G(\tau)$, provided that we represent $G(\tau)$ in the form of a ratio of the boundary values of the canonical functions of the homogeneous problem $Z^+(\tau) / Z^-(\tau)$ in the upper regular D^+ and lower D^- half-plane where

$$Z^{\pm}(\omega) = \frac{1}{N\omega - L \pm i\sqrt{\omega-1}} \quad (\sqrt{\omega-1} = \sqrt{\tau-1} \text{ for } \omega = \tau > 1)$$

since we have $\sqrt{\omega-1} = \pm i\sqrt{1-\tau}$, $\omega = \tau < 1$ when $\omega \in D^{\pm}$. The regular character of the canonical functions $Z^{\pm}(\omega)$ in $D^{\pm}$ follows from the fact that the denomimator of $Z^{\pm}(\omega)$ does not vanish. The final solution has the form

$$\Gamma(\omega) = \frac{Z^+(\omega)}{\pi\sqrt{\omega^2-1}}\left\{\vartheta(M_1-1)\left[\nu_5 \frac{d}{ds}\frac{\sqrt{s^2-1}\,(Ns-L-\sqrt{1-s})}{s-\omega}\right|_{s=\tau_1} - \right.$$

$$\frac{\nu_2\sqrt{\tau_1^2-1}\,(N\tau_1-L-\sqrt{1-\tau_1})}{\tau_1-\omega} +$$

$$\int_{\tau_1}^{-1} \frac{(Ns-L-\sqrt{1-s})\,\Lambda(s)}{\sqrt{-s+\sqrt{s^2-1}}+\sqrt{-s-\sqrt{s^2-1}}-2m}\,\frac{ds}{s-\omega}\Bigg] +$$

$$\vartheta(1-M_1)\left[-\nu_6\frac{d}{ds}\frac{\sqrt{1-s^2}\,(Ns-L-\sqrt{1-s})}{(Ns-L)(s-\omega)}\right|_{s=\tau_2} +$$

$$\frac{\nu_4\sqrt{1-\tau_2^2}\,(N\tau_2-L-\sqrt{1-\tau_2})}{(N\tau_2-L)(\tau_2-\omega)} -$$

$$\frac{m_1}{\sqrt{2}}\int_{\tau_3}^{1}\frac{(Ns-L-\sqrt{1-s})\,\Lambda(s)}{(Ns-L)(\sqrt{2}m-\sqrt{1-s})}\,\frac{ds}{s-\omega}\Bigg] +$$

$$\left.\frac{1}{2}\int_{1}^{\tau_4}\sqrt{\frac{s-1}{s+1}}\,(\sqrt{s+1}+\sqrt{s-1})\,F\left(m_1\sqrt{\frac{s-1}{s+1}}\right)\frac{ds}{s-\omega} + C_0\right\}$$

Here C_0 is a real constant determined from the condition of orthogonality of the perturbed shock wavefront $x = m + \varepsilon\psi(y)$ to the tangential discontinuity $y = \varepsilon f(x + M_1)$ at the point of their intersection.

The pressure

$$p = \operatorname{Im}\int \Gamma(\omega)\,d\omega, \quad p(\infty) = 0$$

has a pole at each point of action of the dipole, and a logarithmic singularity at the point of intersection of the unperturbed shock wavefront with the unperturbed tangential discontinuity.

The function $\psi(y)$ can be determined from the relation at the shock wave

$$y\psi'(y) - \psi(y) = H(y), \quad \psi(m_1) = 0, \quad \psi(y) = y\int_{m_1}^{y} s^{-2}H(s)\,ds$$

$$H(y) = M_1^{-1}\{m^{-1}\vartheta(y_0-y)[Ep^\circ(y) + Gu^\circ(y)] - Bp(y)\}$$

When $V < a_2$, the function $H(y)$ has a logarithmic singularity at the point $y = 0$, and so has $\psi(y)$. The singularity vanishes when $V > a_2$. Since $H(m_1) = 0$, the condition that $\psi(y)$ is smooth at the point $y = m_1$, holds.

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